STATE-SPACE APPROACHES FOR MODELLING REDUCTION OF PILOT SYMBOL ASSISTED MODULATION AND THEIR IMPACT ON THE CHANNEL ESTIMATION

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ABSTRACT

This paper outlines the use of a balance truncation algorithm to design low order infinite impulse response (IIR) interpolator for pilot symbol assisted modulation. A state space model is developed to optimize filter coefficients. The proposed design for the filter has a frequency response is quite stable. This approach is highly beneficial for systems employing fading channel estimation. A comparison is given for the achieved results to that with ideal case.

KEYWORDS

FIR, IIR, Fading channels, PSAM

1. INTRODUCTION

In a world of dramatic changing technology, people need more and more requirements to communicate and get connected with each other and facilitate their life. These approaches may relate with appropriate and timely access to information regardless of the location of people or type of information like scientific collaboration, telemedicine, and real-time environment monitoring. These applications require access to high-bandwidth real time data, images, and video captured from remote sensors such as satellite, radars, and echocardiography [1]. The task at the receiving end of a communication system is to decode the received signal and produce a bit stream that matches the original transmission ones. However, due to the distortion caused by the channel, this process cannot be fulfilled directly. Therefore, a suitable way should be finding to overcome such issue to adjust the received signals before starting the demodulation process. One of the efficient methods is Pilot Symbol Assisted Modulation (PSAM). This method is used to reduce the effects of the fading in mobile communication systems [2-4]. The basic concept of the PSAM is to multiplex training symbols known to the receiver into the data stream. These pilot symbols and the specific multiplexing scheme are known at the receiver and can be exploited for channel estimation, receiver adaptation, and optimal decoding. The receiver uses the pilot symbol to estimate the channel state information (CSI) [5]. In recent years, reduction techniques for linear dynamical systems have drawn a considerable attention. Several interpolation methods have been proposed for PSAM, including low-pass sinc interpolation [2], and optimal Wiener interpolator [6]. Also, there are numerous models have been proposed to analyse the performance in the frequency domain [7]. Moore, in [8], developed a state-space concept based on measurements of controllability and observability in certain directions of the

DOI : 10.5121/ijwmn.2012.4413

state space coordinates. It is known that grammians are not invariant under coordinate transformation. On the other hand, Beliczynski et. al., [9] presented a no minimal realization method for obtaining reduced order model for multivariable system. They have shown that such method encompasses the methods of aggregation (eigenvalue preservation) and moments matching. This paper presents a proposal bases on representing the goal function as a sum of orthonormalized complex decaying exponential. In relative large order the state space concepts in reduction implementation is often simplified, for its small matrix computation requirements. Also, a method to reduce the complexity of the wireless estimator is introduced. This approach is based on an efficient reduction form high order estimator to a low order one with an acceptable tolerance. The paper is organized as follows: Data format is illustrated in section 2 and modelling with Balance truncation is given in section 3 then the obtained results are presented in section 4, finally the conclusion is drawn to section 5.

2. Data format:

The data is formatted into frames of symbols. The first symbol in each frame occupied for the pilot as shown in Fig.1, where the NP is the total number of symbols per frame.

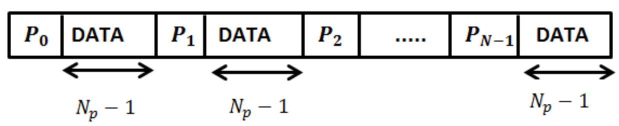

Fig. 1 Frame format

The pilot vectors $\boldsymbol{P}_i$ for i=0, 1,…., N-1 represent the input to the interpolator. The receiver estimates the fading at the i-th data symbol time in the n-th frame from the nearest NP pilot symbols, i.e., the receiver uses $\lfloor (N_P - 1)/2 \rfloor$ pilot symbols from previous frames, the pilot symbol from the current frame, and the pilot symbols $\lfloor N_P/2 \rfloor$ from the subsequent frames is illustrated in Fig.2. Thus, the estimated fading is given by:

$$\tilde{z}_n{}^i = \sum_{k=-\lfloor (N_P-1)/2 \rfloor}^{\lfloor N_P/2 \rfloor} f_k{}^i P_k \qquad (1)$$

where $f_k{}^i$ is the coefficient of the interpolator.

Sinc interpolation is used in this work due to its simplicity with the PSAM. The interpolation coefficients are computed from the sinc function as:

$$f_k{}^i = sinc(\frac{i}{N_p} - k) \qquad (2)$$

where $k = -\lfloor (\mathrm{N_P} - 1)/2 \rfloor, \ldots, \lfloor \mathrm{N_P}/2 \rfloor$. The sinc interpolator acts finite impulse response filter (FIR). FIR filters are simple to design and are guaranteed to be bounded input-bounded output (BIBO) stable. By designing the filter taps to be symmetrical about the centre tap position, the FIR filters have linear phase characteristics. The principle that is required throughout the pass-band of the filter response to preserve the shape of a given signal within the pass-band. Such property is desired for many applications such as in music and video processing [10].

Another type of the digital filter is (IIR filter) which occupies much lower implementation complexity than the FIR filter since it appears with a smaller order[10]. IIR filters suffer from a nonlinear-phase response, which limits their applications in 1-D system. To compensate this phase nonlinearly some additional phase equalizations circuit are required. This intern will add some extra complexity to the system implementation.

Usually the problem of approximation of a given system by lowering the order (requiring fewer memory elements system) has been surfaced in signal processing and control literature [9,11,12]. So, the yielding system after approximation support suppressing the complexity with keeping the same magnitude and phase responses as those of high order system (full order system) which are often desirable in practice. The linear model order reduction problem is given as "a Linear time invariant" (LTI) system G(z) of order N > n is given. It is desired to find an equivalent LTI system $\tilde{G}(z)$ of order no greater than n such that $\left\| G(z) - \tilde{G}(z) \right\|$ is as small as possible, where $\|.\|$ denotes some distance given by [8] and [2].

For

$$G(z) - \tilde{G}(z) = e(z) = \sum_{i=0}^{n} \varepsilon_i z^{-i} \tag{3}$$

where

$$\varepsilon_i = \frac{1}{2\pi j} \oint_c e(z).z^{i-1} dz \tag{4}$$

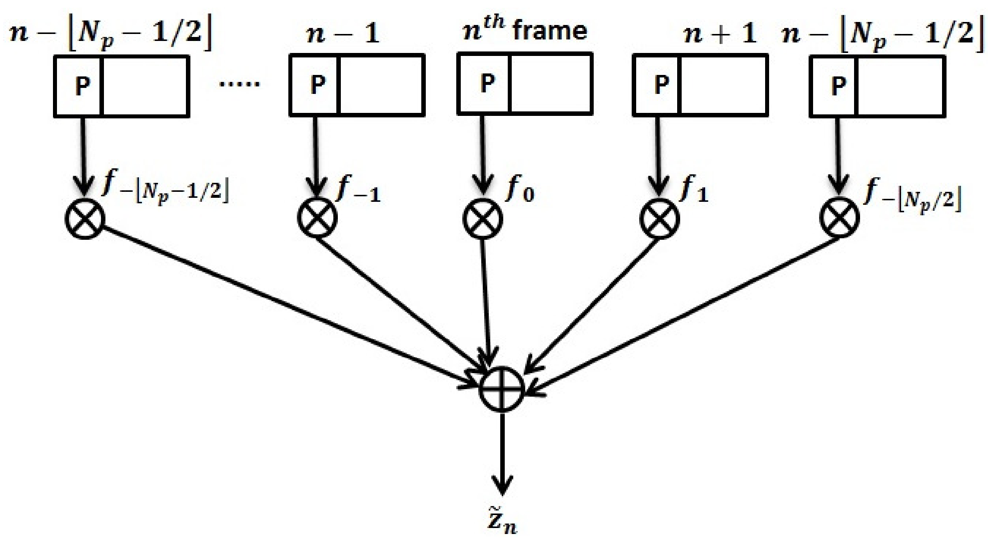

Fig. 2 Fading calculation in PSAM.

3. Balanced realization and reduction model:

3.1 Balanced realization:

The method of balanced realization is widely used in the context of realization theory for model of reduction construction of linear systems [11, 13]. This approach requires only standard matrix computations. The state-space description of a stable system is transformed to balanced coordinate via nonsingular matrix tool (T) satisfies that the observability and controllability grammians are equal and diagonal (balanced) [11, 12, 13]. Let $(A, B, C, D)_N$ represents a full-order 1-D dynamic stable system described by[12]:

$$\begin{aligned} x(k+1) &= \mathbf{A}\, x(k) + \mathbf{B}\, u(k) \\ y(k) &= \mathbf{C}\, x(k) + \mathbf{D}\, u(k) \end{aligned} \tag{5}$$

where N is the order of realization with z-domain transfer function F(z) that is:

$$F(z) = \boldsymbol{C}(ZI - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D} \tag{6}$$

The controllability and observability grammians are then defined, receptively, as [15]:

$$\boldsymbol{W}_c = \sum_{k=0}^{\infty} \boldsymbol{A}^T \boldsymbol{B}\boldsymbol{B}^T (\boldsymbol{A}^T)^k \tag{7}$$

and

$$\boldsymbol{W}_o = \sum_{k=0}^{\infty} \boldsymbol{A}^T \boldsymbol{C}\boldsymbol{C}^T \boldsymbol{A}^T \tag{8}$$

The grammians are the unique positive defined solution for the Lyapunov equations as [14]:

$$\boldsymbol{A}\boldsymbol{W}_c + \boldsymbol{W}_c\boldsymbol{A}^T = -\boldsymbol{B}\boldsymbol{B}^T \tag{9}$$

then,

$$\boldsymbol{A}^T\boldsymbol{W}_o + \boldsymbol{W}_o\boldsymbol{A} = -\boldsymbol{C}^T\boldsymbol{C} \tag{10}$$

Under nonsingular state transfer matrix via T.

$$F(z) = \bar{\boldsymbol{C}}(ZI - \bar{\mathbf{A}})^{-1}\bar{\boldsymbol{B}} + \bar{\boldsymbol{D}} \tag{11}$$

where,

$$\bar{\boldsymbol{A}} = \boldsymbol{T}^{-1}\boldsymbol{A}\boldsymbol{T}, \bar{\boldsymbol{B}} = \boldsymbol{T}^{-1}\boldsymbol{B}, \quad \bar{\boldsymbol{C}} = \boldsymbol{C}\boldsymbol{T} \text{ and } \bar{\boldsymbol{D}} = \boldsymbol{D} \tag{12}$$

3.2 Hankel operator:

Over the past few decades, the Hankel operator has received a great deal of attention due to its vital role in control and filter model reduction [15]. Consider the proper transfer function:

$$G(z) = \frac{N(z)}{D(z)} = \frac{\beta_o + \beta_1 z^{-1} + \ldots + \beta_n z^{-n}}{\alpha_o + \alpha_1 z^{-1} + \ldots + \alpha_n z^{-n}} \tag{13}$$

where β_i's and α_i's (i=0,1,2,…,n) are constants. It can be expand into an infinite power series of descending power of z^{-1} as:

$$G(z) = c_0 + c_1 z^{-1} + c_2 z^{-2} + \ldots \tag{14}$$

where the coefficients $\{c_i, i = 0,1,2,\ldots\}$ are called Markov parameters. These parameters can be obtained recursively form A, B, C, and D of its state-space representation as:

$$c_i = CA^{i-1} \quad for \ \ i = 1,2,3,\ldots \tag{15}$$

and

$$D = c_0 \tag{16}$$

We define an infinite block-Hankel matrix, and denoted by $\Gamma\{G(z)\}$, as

$$\Gamma\{G(z)\} = \begin{bmatrix} c_1 & c_2 & c_3 & \ldots \\ c_2 & c_3 & \ldots & \\ c_3 & c_4 & \ldots & \\ & & . & \\ & & . & \end{bmatrix} \tag{17}$$

$\Gamma\{G(z)\}$ is formed from coefficients, representing the impulse response. According to Kronecker's theorem [8], the proper form of the transfer function G(z) has degree n if and only if:

$$R[\,\Gamma G(z)\}] = \deg[\,G(z)] \tag{18}$$

The Hankel matrix of Eqn. (17) can be written as:

$$\Gamma\{G(z)\} = QP \tag{19}$$

where P and Q are the controllability and observability matrices, respectively. And defined in state-space coordinates (A, B, C) as [8]:

$$P=[B: AB : \ldots : A^kB : \ldots] \tag{20}$$

$$Q=[C^T : A^TC^T : \ldots : (A^T)^kC^T : \ldots]^T \tag{21}$$

Since the solution of Eqn. (9) can be written as:

$$W_c=BB^T + ABB^TA^T + \ldots + A^kBB^T(A^T)^k + . \tag{22}$$

then

$$W_c= QP^T \tag{23}$$

and the similar one can show that:

$$W_o= Q^TP \tag{24}$$

The Hankel singular values of G(z) are the singular values of Γ{G(z)} and following holds true [15]:

[The i^{th} singular value of $\Gamma\{G(z)\}]^2$ = The i^{th} singular value of the product

$W_c W_o$ = [The i^{th} singular value of $G(z)]^2$ (25)

The filter G(z) may be represented as a set of differences equations as given in (5)

where,

$$A=\begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \\ & & \vdots & & \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix},$$

$$B=\begin{bmatrix} 1 \\ \vdots \\ 0 \end{bmatrix}, C=\begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix},$$

$$D=\begin{bmatrix} c_o \end{bmatrix} \tag{26}$$

Notice that for $(A, B, C)_N$ system having a finite impulse response, the rows and columns of zeros are omitted and the following finite Hankel matrix is used :

$$H = \begin{bmatrix} c_1 & c_2 & \cdots & 0 & c_N \\ c_2 & c_3 & \cdots & c_N & 0 \\ c_3 & c_4 & \cdots & 0 & 0 \\ & & \vdots & & \\ c_N & 0 & \cdots & 0 & 0 \end{bmatrix} \tag{27}$$

The matrices W_c and W_o, defined by Eqns. (7) and (8) have the same dimension of Hand W_c , $W_o \in R_{NxN}$

where R_{NxN} is the set of real matrices.

The H matrix (27) is a symmetric matrix so it can be decomposed (SVD-decomposition) to:

$$H=V\Lambda V^T \tag{28}$$

where

$$VV^T=I \tag{29}$$

and Λ is a diagonal matrix with the eigenvalues of H (Hankel singular values), and I is a unit matrix. Notice that H is not necessary to be a positive-definite matrix.

For the system $(A, B, C)_N$ where A, B, and C matrices are given in Eqn. (26), the controllability matrix, and controllability grammians are unit matrices , i.e.,

$$P = I \tag{30}$$

$$W_c=I \tag{31}$$

and the transformation matrix tool

$$T = V \left| \Lambda \right|^{1/2} \tag{32}$$

with lead to balanced realization of the system Eqn. (26) where V and Λ are defined by Eqns. (28) and (29), respectively. And, $|.|$ denotes the absolute value of the matrix elements.

3.3 Balanced truncation:

Considering the fact that the full-order filter of Eqn.(26) is optimal. An optimal reduced one should be close to the optimal one form the input-output of view. Furthermore, it is known from the applied the transformation process on the (A, B, C)N by (T) according to Eqn. (31) results in a balanced system ($\overline{\mathbf{A}}, \overline{\mathrm{B}}, \overline{\mathrm{C}}$)N.

Next, assume that the matrices Λ, andV can be decomposed each on into two parts as:

$$\Lambda = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \tag{33}$$

where

$$\Lambda_1 = diag(\nu_1, \nu_2, ..., \nu_n) \tag{34}$$

and

$$\Lambda_2 = diag(\nu_{n+1}, \nu_{n+2}, ..., \nu_N) \tag{35}$$

$$|v_i| = |\sigma_i| \tag{36}$$

where σ_i's are the singular values of F(z)and

$$V = [V_1 \qquad V_2 \;] \tag{37}$$

where V_1, and V_2 are (N x n), and (N x N-n) rectangular matrices, respectively.

According to the partition, the balanced system coordinates can be represented as

$$\overline{A} = \begin{bmatrix} \overline{A}_{11} & \overline{A}_{12} \\ \overline{A}_{21} & \overline{A}_{22} \end{bmatrix},\ \overline{B} = \begin{bmatrix} \overline{B}_1 \\ \overline{B}_2 \end{bmatrix}, C = \begin{bmatrix} \overline{C}_1 \\ \overline{C}_2 \end{bmatrix} \tag{38}$$

where

$$\overline{A}_{11} = |\Lambda_1|^{1/2} V_1^T A V_1 |\Lambda_1|^{-1/2} \tag{39}$$

$$\overline{A}_{12} = |\Lambda_1|^{1/2} V_1^T A V_2 |\Lambda_2|^{-1/2} \tag{40}$$

$$\overline{A}_{21} = |\Lambda_2|^{1/2} V_2^T A V_1 |\Lambda_1|^{-1/2} \tag{41}$$

$$\overline{A}_{22} = |\Lambda_2|^{1/2} V_2^T A V_2 |\Lambda_2|^{-1/2} \tag{42}$$

$$\overline{B}_1 = |\Lambda_1|^{1/2} V_1^T B \tag{43}$$

$$\overline{B}_2 = |\Lambda_2|^{1/2} V_2^T B \tag{44}$$

and

$$\overline{C}_1 = C\, V_1 |\Lambda_1|^{-1/2} \tag{45}$$

$$\overline{C}_2 = C\, V_2 |\Lambda_2|^{-1/2} \tag{46}$$

The orthogonal matrix V achieves that Eqn. (43) is always existed. The full order system (A, B, C)$_N$ is asymptotically stable. Then, the following two-system found to be asymptotically stable and balanced:

(i) The truncated system - $(\bar{A}_{11},\bar{\boldsymbol{B}}_{\boldsymbol{1}},\bar{\boldsymbol{C}}_{\boldsymbol{1}})$.

(ii) (ii) The rejected system - $(\bar{A}_{22}, \bar{\boldsymbol{B}}_{\boldsymbol{2}}, \bar{\boldsymbol{C}}_{\boldsymbol{2}})$.

If $\tilde{G}(z)$ is a transfer function obtained by truncating the balanced realization of G(z) to the first n states (if σ 's values of the grammian Σ are ordered in a descending manner whereas if σ's order in a sending fashion, the truncated system $\tilde{G}(z)$ obtained from the last n states then

$$\left\| G(z) - \tilde{G}(z) \right\|_H \leq 2tr(\ \Sigma_2\ ) \tag{47}$$

The result (47) is presented in [15] where tr (.) is the trace of a matrix.

In inequality (47), the Hankel norm used because it compromises between two conventional-norms: Euclidean norm and the Chebyshev norms.

There is an error in estimation of fading due to the difference between the ideal and approximated one. The impact of this error mirror on the overall channel estimation. This study until now don't take in account certain transmission system. If single user multi-level QAM (MQAM) is taken, the impact of this interpellator appears in the calculation of received signal to noise ratio (SNR).The instantaneous BER is now given by[17]:

$$BER \leq 0.2\ e^{-1.5\,\bar{\gamma}/(M-1)} \tag{48}$$

where BER , $\bar{\gamma}$ and M are the bit error rate, average approximated SNR and modulation level, respectively.

The spectral efficiency for fixed M is $\log_2$, the number of bits/symbol.

4 Results:

Numerical results are presented to demonstrate the analyses in the preceding sections. Below sample of responses of the approximated IIR estimators for 3, and 6 level truncation with respect to 0 truncation (without truncation). The magnitude and group delay responses of the low truncation level are closed to the responses of FIR one. The impact on the estimation in this case may be neglected. In the case of high level truncation the responses either for magnitude or phase (or group delay is the derivative of the phase function w.r.t. radian frequency) are differ, so the impact be high on the channel fading parameters estimation (magnitude and phase). In order to compensate this effect may be treat it by using efficient channel coding scheme but this will spend portion from the spectral of the channel. Figure 1 and 2 denotes to the exact and approximate magnitude and group delay of 3_{rd} level and 6^{th} level truncation, respectively. The error norm between the two responses is shown in Figure 3.

5 Conclusion

The paper presented a method to reduce the order of interpolator of PSAM. There is a closed-form solution for the observability grammian and Hankel singular values of FIR filters. The balanced realization technique is then applied to find a reduced-order IIR model with a closed-form bound for the infinity norm of the approximation error. The resultant error bound depends on the order of the original IIR filter. Conditions under which the proposed technique leads to a sufficiently small approximation error are given.

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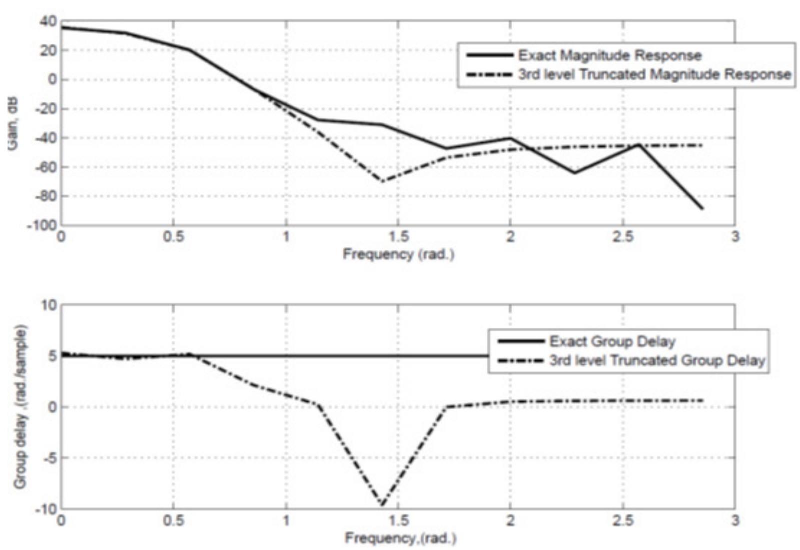

Figure 1: Magnitude response (in dB.) and group delay (in rad./sample) of the approximated IIR estimator for truncation level=3

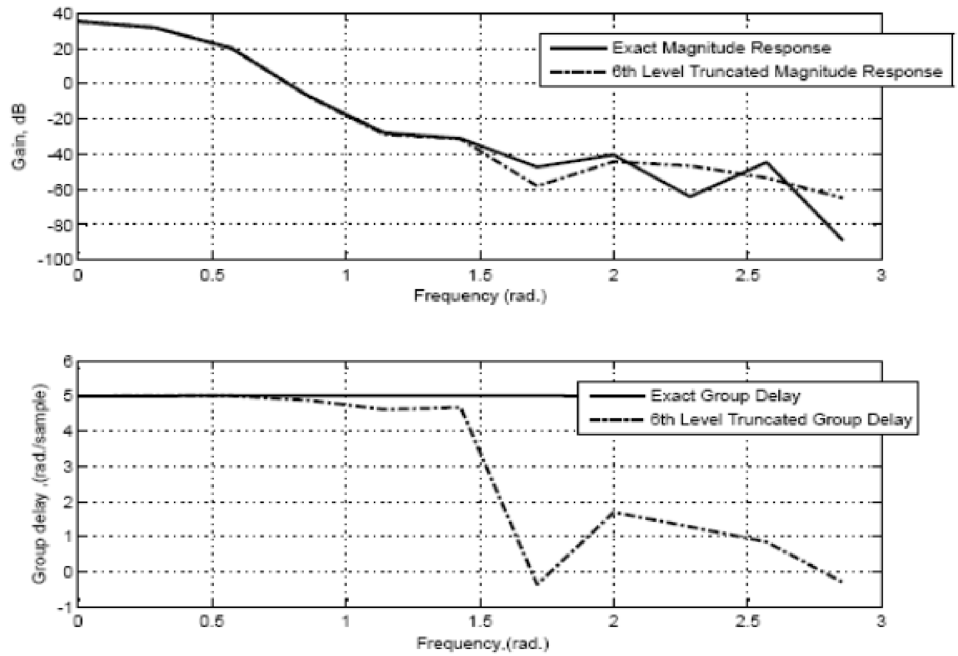

Figure 2: Magnitude response (in dB.) and group delay (in rad./sample) of the approximated IIR estimator for truncation level=6

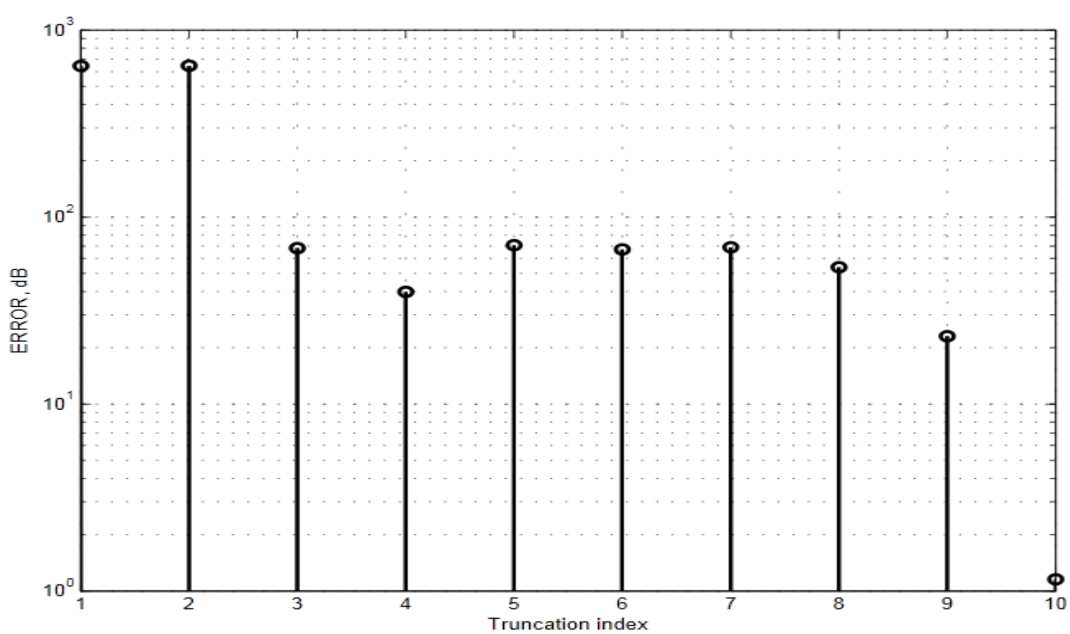

Figure 3: Truncation index versus the error in (dB.)