HYPERCHAOS SYNCHRONIZATION USING GBM

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ABSTRACT

This paper presents synchronization of a new autonomous hyperchaotic system. The generalized backstepping technique is applied to achieve hyperchaos synchronization for the two new hyperchaotic systems. Generalized backstepping method is similarity to backstepping. Backstepping method is used only to strictly feedback systems but generalized backstepping method expands this class. Numerical simulations are presented to demonstrate the effectiveness of the synchronization schemes.

KEYWORDS

New hyperchaotic system, Generalized backstepping method, Synchronization.

1. INTRODUCTION

In recent years, chaos and hyperchaos generation, control and synchronization has become more and more interesting topics to engineering. Robust Control [1], the sliding method control [2], linear and nonlinear feedback control [3], adaptive control [4], active control [5], backstepping control [6] and generalized backsteppig method control [7-9] are various methods to achieve the synchronization hyperchaotic systems.

Active control and Lyapunov theory has been used to anti-synchronized for the identical and non-identical hyperchaotic Qi and Jha systems [10]. [11] has used sliding mode control to global synchronized for the identical Qi, Liu and Wang four-wing chaotic systems. [12] derived new results for the global chaos synchronization for the Sprott-L and Sprott-M systems using active control method and we established the synchronization results with the help of Lyapunov stability theory. In [13], active control method was applied to derive anti-synchronization results for the identical hyperchaotic Li systems (2005), identical hyperchaotic Lü systems (2008), and nonidentical hyperchaotic Li and hyperchaotic Lü systems. [14] has deployed sliding control to achieve global chaos synchronization. [15] has applied active control for the derivation of state feedback control laws so as to global anti-synchronized of identical and non-identical hyperchaotic Liu and Qi systems. [16] has used sliding mode control to achieve global chaos synchronization for the Chen systems. The adaptive generalized backstepping method was applied to control of uncertain Sprott-H chaotic system in [17].

The rest of the paper is organized as follows: In section 2, a novel hyperchaos is presented. In section 3, the generalized backstepping method is studied. In section 4, synchronization between two new hyperchaotic systems are achieved by generalized backstepping control. In section 5, Represents simulation results. Finally, in section 6, Provides conclusion of this work.

2. SYSTEM DESCRIPTION

Recently, Ling Liu et al constructed the new hyperchaotic system [18]. The system is described by.

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= bx + xz - w \\ \dot{z} &= -xy - cz + w \\ \dot{w} &= dx + y \end{aligned} \tag{1}$$

Where a, b, c, d are positive constants and x, y, z, w are variables of the system, when $a = 10, b = 35, c = 1{,}4, d = 5$, the system (1) is hyperchaotic. See Figure 1 and Figure 2.

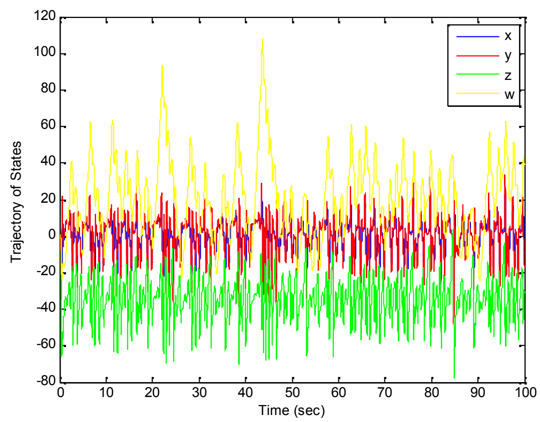

Figure 1. Time response of the system (1).

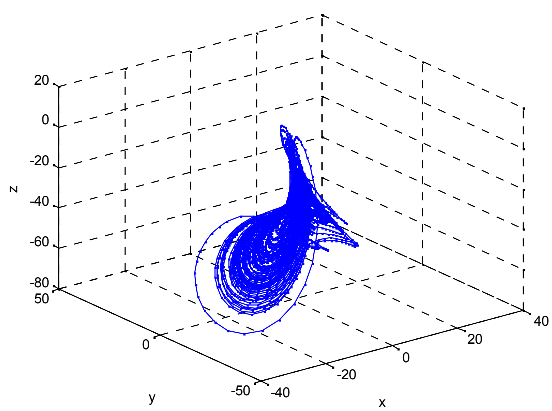

Figure 2. Phase portraits of the hyperchaotic attractors (1).

3. GENERALIZED BACKSTEPPING METHOD

Generalized Backstepping Method [7-9] is applied to nonlinear systems as follow

$$\begin{cases} \dot{X} = F(X) + G(X)\eta \\ \dot{\eta} = f_0(X,\eta) + g_0(X,\eta)u \end{cases} \tag{2}$$

Where $\eta \epsilon$□ and $x = [x_1, x_2, \cdots, x_n] \epsilon$□. The Lyapunov function are supposed as follow

$$V(X) = \frac{1}{2}\sum_{i=1}^{n} x_i^2 \tag{3}$$

The control signal and the extended lyapunov function of system (2) are obtained by equations (4),(5).

$$u = \frac{1}{g_0(X,\eta)}\left\{ \begin{array}{c} \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial \varphi_i}{\partial x_j}[f_i(X) + g_i(X)\eta] \\ -\sum_{i=1}^{n} x_i g_i(X) - \sum_{i=1}^{n} k_i[\eta - \varphi_i(X)] - f_0(X,\eta) \end{array} \right\}, k_i > 0, i = 1,2,\cdots,n \tag{4}$$

$$V_t(X,\eta) = \frac{1}{2}\sum_{i=1}^{n} x_i^2 + \frac{1}{2}\sum_{i=1}^{n}[\eta - \varphi_i(X)]^2 \tag{5}$$

4. SYNCHRONIZATION OF HYPERCHAOTIC SYSTEM

In this section, the generalized backstepping method is applied to synchronize two identical hyperchaotic systems with known parameters and some global asymptotic synchronization conditions are obtained. The master system is as follows

$$\begin{aligned} \dot{x}_1 &= a(y_1 - x_1) \\ \dot{y}_1 &= bx_1 + x_1 z_1 - w_1 \\ \dot{z}_1 &= -x_1 y_1 - cz_1 + w_1 \\ \dot{w}_1 &= dx_1 + y_1 \end{aligned} \tag{6}$$

and the slave system is as follows

$$\begin{aligned} \dot{x}_2 &= a(y_2 - x_2) \\ \dot{y}_2 &= bx_2 + x_2 z_2 - w_2 + u_1(t) \\ \dot{z}_2 &= -x_2 y_2 - cz_2 + w_2 + u_2(t) \\ \dot{w}_2 &= dx_2 + y_2 \end{aligned} \tag{7}$$

Where $u_1(t), u_2(t)$ are control functions to be determined for achieving synchronization between the two systems (6) and (7). Define state errors between systems (6) and (7) as follows

$$\begin{aligned} e_x &= x_2 - x_1 \\ e_y &= y_2 + y_1 \\ e_z &= z_2 - z_1 \\ e_w &= w_2 + w_1 \end{aligned} \tag{8}$$

We obtain the following error dynamical system by subtracting the drive system (6) from the response system (7)

$$\begin{aligned} \dot{e}_x &= a(e_y - e_x) \\ \dot{e}_y &= be_x - e_w + (x_2 z_2 - x_1 z_1) + u_1(t) \\ \dot{e}_z &= -ce_z + e_w - (x_2 y_2 - x_1 y_1) + u_2(t) \\ \dot{e}_w &= de_x + e_y \end{aligned} \tag{9}$$

Define the following active control functions $u_1(t), u_2(t)$

$$\begin{aligned} u_1(t) &= x_1 z_1 - x_2 z_2 + v_1(t) \\ u_2(t) &= x_2 y_2 - x_1 y_1 + v_2(t) \end{aligned} \tag{10}$$

Where $v_1(t), v_2(t)$ are control inputs. Substituting equation (10) into equation (9) yields

$$\begin{aligned} \dot{e}_x &= a(e_y - e_x) \\ \dot{e}_y &= be_x - e_w + v_1(t) \\ \dot{e}_z &= -ce_z + e_w + v_2(t) \\ \dot{e}_w &= de_x + e_y \end{aligned} \tag{11}$$

Thus, the error system (11) to be controlled with control inputs $v_1(t), v_2(t)$ as functions of error states e_x, e_y, e_z and e_w. When system (11) is stabilized by control inputs $v_1(t), v_2(t)$, e_x, e_y, e_z and e_w will converage to zeroes as time t tends to infinity. Which implies that system (6) and (7) are synchronized. To achieve this purpose, we choose control inputs by using generalized backstepping control such that

$$\begin{aligned} v_1(t) &= -d\dot{e}_x - k_1\dot{e}_w - ae_w - k_2(e_y - \varphi_1) - k_3(e_y - \varphi_2) - be_w \\ v_2(t) &= -e_w - k_4 e_z \end{aligned} \tag{12}$$

Where

$$\begin{aligned} \varphi_1(e_x, e_y, e_z, e_w) &= 0 \\ \varphi_2(e_x, e_y, e_z, e_w) &= -de_x - k_1 e_w \end{aligned} \tag{13}$$

we select the gains of controllers (12) in the following form

$$k_1 = 10, k_2 = 9, k_3 = 10, k_4 = 8 \tag{14}$$

And Lyapunov function as

$$V(e_x, e_y, e_z, e_w) = \frac{1}{2}e_x^2 + \frac{1}{2}e_y^2 + \frac{1}{2}e_z^2 + \frac{1}{2}e_w^2 + \frac{1}{2}(e_y - \varphi_1)^2 + \frac{1}{2}(e_y - \varphi_2)^2 \tag{15}$$

5. NUMERICAL SIMULATION

The generalized backstepping control is used as an approach to synchronize two new hyperchaotic systems. The initial values are $x_1(0) = -9, y_1(0) = 5, z_1(0) = -6, w_1(0) = 7$ and $x_2(0) = 8, y_2(0) = -7, z_2(0) = 8, w_2(0) = -9$ respectively. The time response of x, y, z, w states for drive system (6) and the response system (7) via generalized backstepping method shown in Figure 3. Synchronization errors (e_x, e_y, e_z, e_w) in the hyperchaotic systems shown in Figure 4. The time response of the control inputs (u_1, u_2) for the synchronization hyperchaotic systems shown in Figure 5.

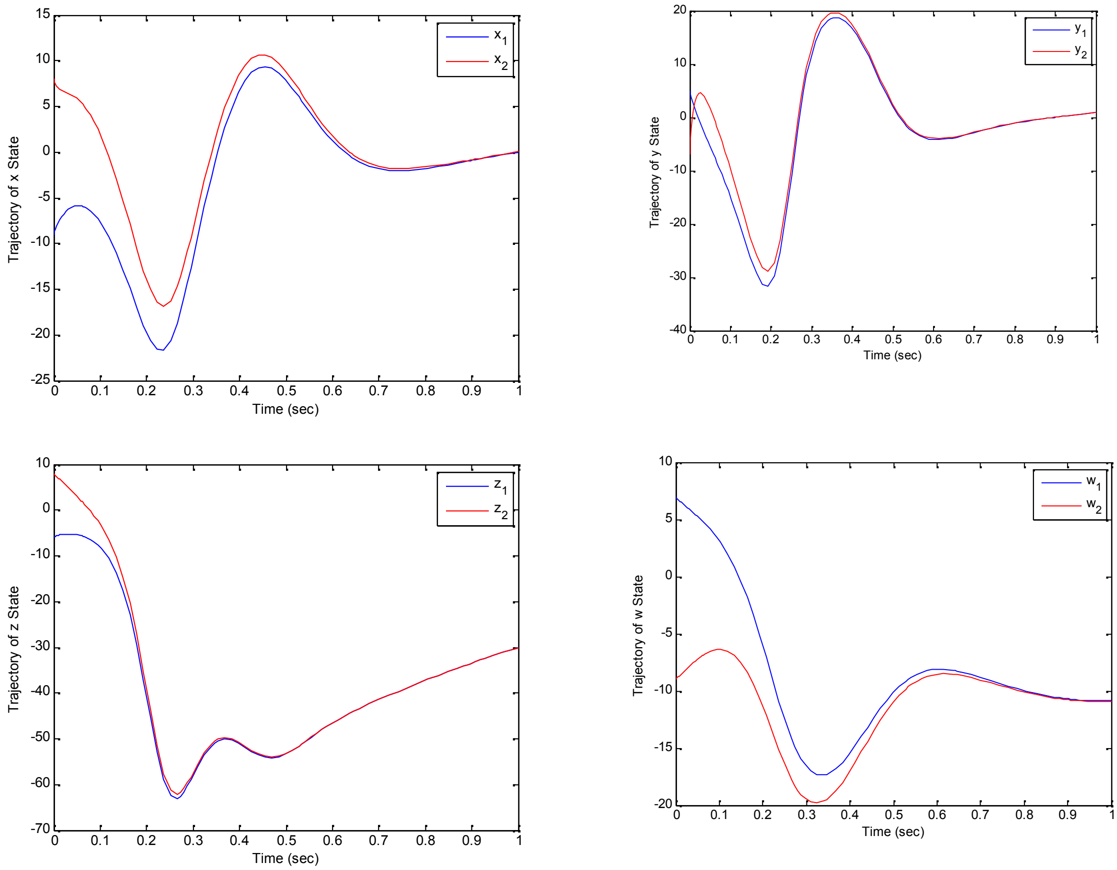

Figure 3. The time response of signal (x, y, z, w) for drive system (6) and response system (7).

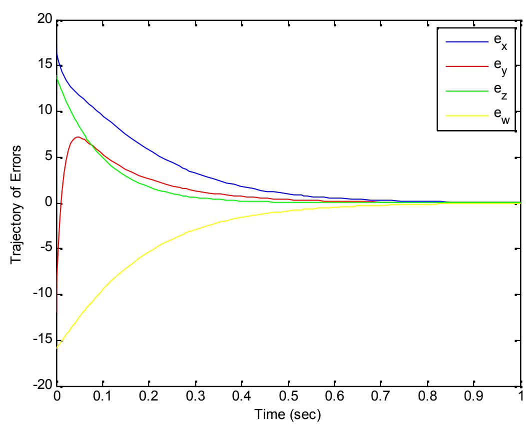

Figure 4. Synchronization errors (e_x, e_y, e_z, e_w) in drive system (6) and response system (7).

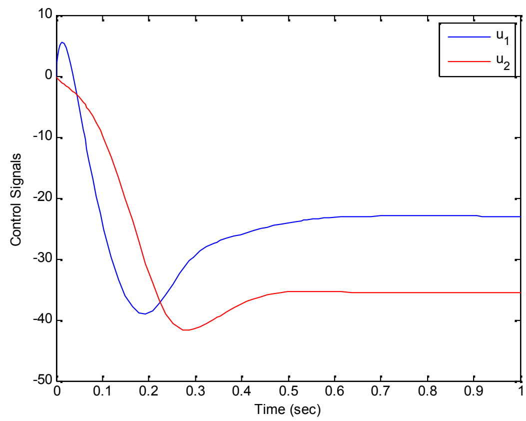

Figure 5. The time response of the control inputs (u_1, u_2) for drive system (6) and response system (7).

6. CONCLUSIONS

We investigate chaos synchronization of a new autonomous hyperchaotic system. This synchronization between two new systems was achieved by generalized backstepping method. Based on the generalized backstepping method, corresponding controller is designed to achieve synchronization between two identical new hyperchaotic systems. Numerical simulations show that the proposed method work effectively.

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